A Sound and Complete Abstraction for Reasoning about Parallel Prefix Sums

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The interval of summations is a novel abstraction for reasoning about parallel prefix sums. With it, the correctness of any generic prefix sum implementation can be established by checking a single test case.

1. Prefix sums

The prefix sum for an associative binary operator \( \oplus \) takes \([ s_1, s_2, \ldots, s_n ]\) and returns \([ s_1, s_1 \oplus s_2, \ldots, s_1 \oplus s_2 \oplus \cdots \oplus s_n ]\), the list of all prefixes.

2. Examples and Utility

Prefix sums have been extensively studied in hardware and parallel software design for their utility in applications such as carry-lookahead adders, stream compaction, and sorting algorithms.

Here are circuits for four well-known prefix sums:

Kogge-Stone
Sklansky
Brent-Kung
Blelloch

3. The Interval of Summations

We observe that a prefix sum algorithm may only exploit the property of associativity.

Abstract a concrete summation \( s_i \oplus s_{i+1} \oplus \cdots \oplus s_j \) by the abstract interval \((i,j)\).

Define the sum of kissing intervals by \((i,j) \oplus (k,l) = (i,l)\) if \(j + 1 = k\).

The sum of non-kissing intervals is \(\top\).

This abstraction allows us to establish the correctness of any prefix sum by running the implementation on the input \([ (1,1), (2,2), \ldots, (n,n) ]\) and checking that it computes the output \([ (1,1), (1,2), \ldots, (1,n) ]\). We then extend this result to a data-parallel setting.

Our paper and talk

Read our paper for theoretical and practical results, which show the power and utility of this custom abstraction.

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Session 5b Reasoning 3’15pm

http://multicore.doc.ic.ac.uk/tools/GPUVerify/POPL14

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