A Sound and Complete Abstraction for Reasoning about Parallel Prefix Sums

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The **interval of summations** is a novel abstraction for reasoning about parallel prefix sums. With it, the correctness of any generic prefix sum implementation can be established by checking a single test case.

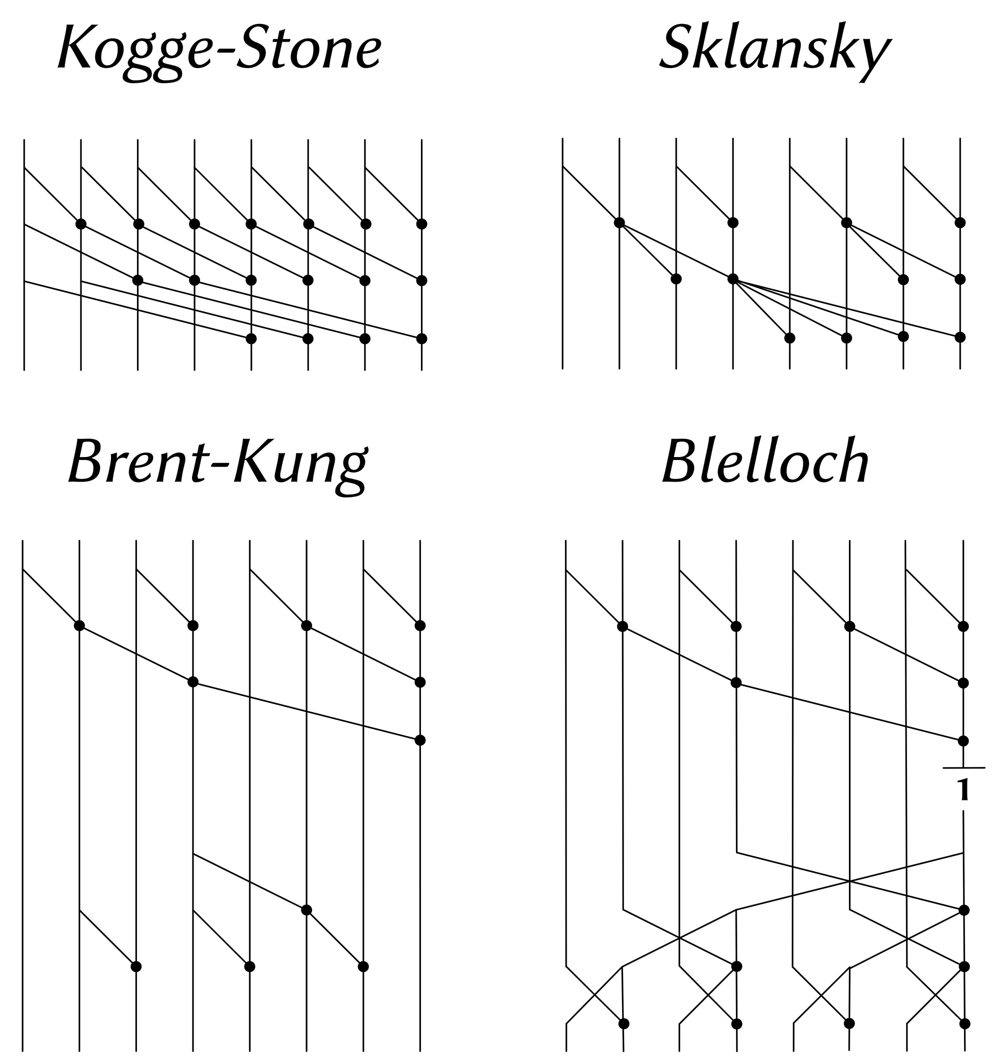
1. Prefix sums

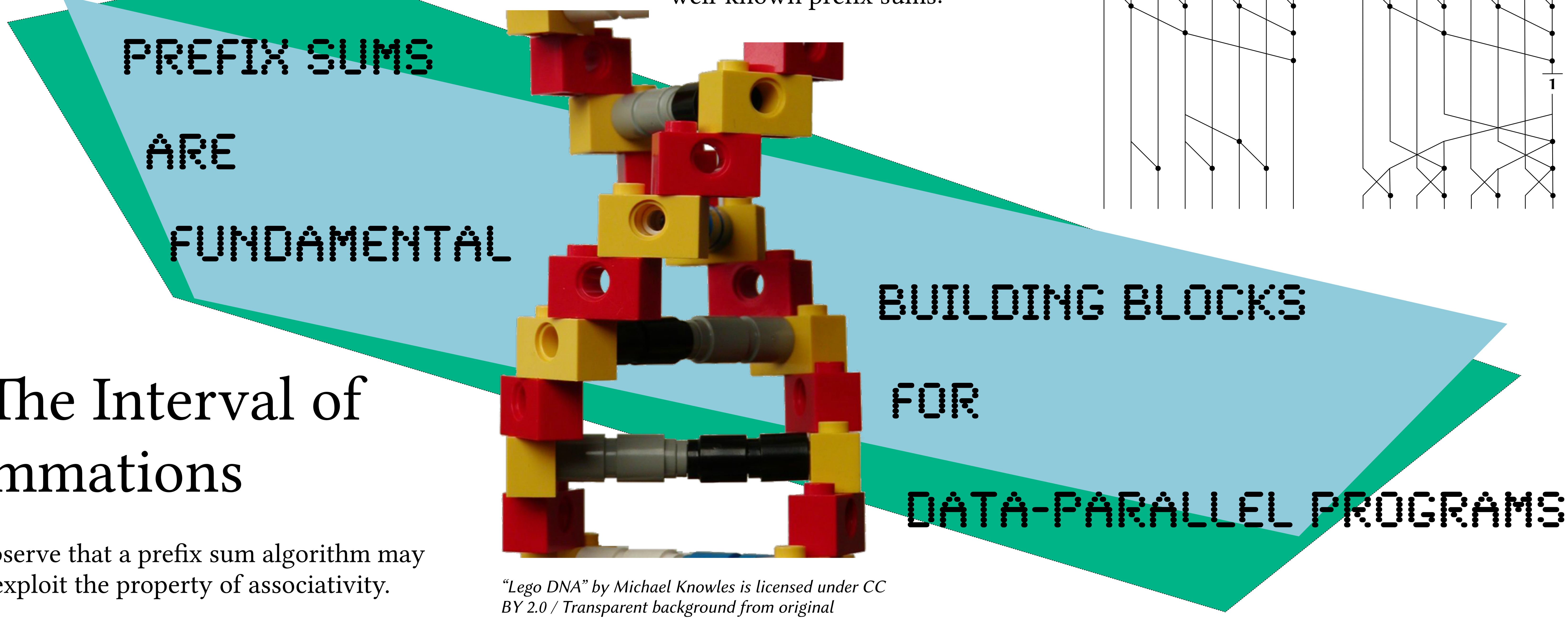
The prefix sum for an associative binary operator $\oplus$ takes [$s_1, s_2, \ldots, s_n$] and returns [$s_1, s_1 \oplus s_2, \ldots, s_1 \oplus s_2 \oplus \cdots \oplus s_n$], the list of all prefixes.

2. Examples and Utility

Prefix sums have been extensively studied in hardware and parallel software design for their utility in applications such as carry-lookahead adders, stream compaction, and sorting algorithms.

Here are circuits for four well-known prefix sums:





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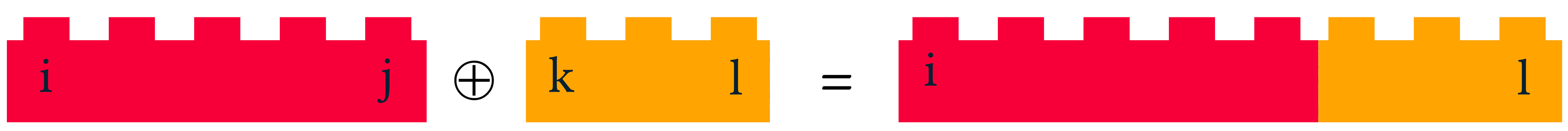
3. The Interval of Summations

We observe that a prefix sum algorithm may **only** exploit the property of associativity.

Abstract a concrete summation $s_i \oplus s_{i+1} \oplus \cdots \oplus s_j$ by the abstract interval (i,j)
Define the sum of kissing intervals by $(i,j) \oplus (k,l) = (i,l)$ if $j + 1 = k$.
The sum of non-kissing intervals is $\top$.



This abstraction allows us to establish the correctness of any prefix sum by running the implementation on the input [(1,1), (2,2) ..., (n, n)] and checking that it computes the output [(1,1), (1,2), ..., (1, n)]. We then extend this result to a data-parallel setting.

Our paper and talk

Read our paper for theoretical and practical results, which show the power and utility of this custom abstraction.

Thursday 23rd January (Day 2)
Session 5b Reasoning 3'15pm

http://multicore.doc.ic.ac.uk/tools/GPUVerify/POPL14

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